

**Doc  
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MPHYS NOTES

**Introduction to Nonlinear  
Physics**

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*Based on a course by Prof. Anne Juel*

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# Chapter 1

## Introduction

1.1 Introduction and Motivation

1.2 Examples of Linearity and Nonlinearity  
in Science

1.3 Modelling Systems Using Iterated Maps  
and Differential Equations



## Chapter 2

# General Features of Dynamical Systems

2.1 Systems of Differential Equations

2.2 Control Parameters

2.3 Fixed Points and Their Stabilities

2.4 Phase Space

2.5 Linear Stability Analysis

2.6 Numerical Methods for Nonlinear Systems

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# Chapter 3

## The Logistic Map

- 3.1 Linear and Quadratic Maps
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- 3.3 Linear Stability Analysis and the Existence of 2-Cycles
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### 4.2 Artificial Fractals

#### 4.2.1 The Cantor Set

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### 4.3 Fractal Dimensions

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5.2 How Strange Attractors Come into Existence

5.3 Evolution of Phase Space Volumes in Chaotic and Non-Chaotic Systems

5.4 Mixing and Information Entropy