

**Doc
Dave**

MPHYS NOTES

Lagrangian Dynamics

Dave Scott

Based on a course by Prof. Terry Wyatt

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1.1.2 External Forces

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Lagrangian Dynamics

- 2.1 The Energy Method Plus Other Conservation Laws
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- 2.3 Small Oscillations and Normal Modes

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6.1 Normal Modes from Symmetries

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- 7.1 Lagrangian for Charged Particles Moving in Electric and Magnetic Fields
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